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A Global Optimal Formulation for the Water Integration in Eco-Industrial Parks Considering Multiple Pollutants

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Abstract

A mathematical programming formulation for the water integration in eco-industrial parks considering streams with several pollutants is presented. The formulation is based on a superstructure that allows the wastewater reuse in the same plant, the water exchange with different plants, and a shared set of interceptors that must be selected to determine the network configuration that satisfies process equipments and environmental constraints. The model formulation considers wastewater with several pollutants, and optimizes the network according to the minimum total annual cost, which includes the costs of fresh water, piping and regeneration. A new discretization approach is also proposed to handle the large set of bilinear terms that appear in the model in order to yield a near global optimal solution. The results obtained in several examples show considerable savings with respect to the solutions of the individual plant integration policy commonly employed for these types of problems.

**Keywords**: water integration, eco-industrial parks, recycle/reuse networks, optimization, convex discretization, inter-plant water integration.
1. Introduction

For all production processes raw materials are processed and transformed into goods or services; however, these processes generate significant amounts of waste streams that are discharged to the environment. In this regard, the increase in the cost of raw materials due to the scarcity of natural resources as well as the stricter environmental constraints have promoted the research efforts for the minimization of production costs and the minimization of the environmental impact. Water, a vital fluid for mankind, is one of the resources most widely used in industry; the design of mass exchange networks based on water, therefore, plays an important role for both social and economic aspects. The synthesis of water networks has been studied from different perspectives (i.e., single plant integration, inter-plant integration, minimization of freshwater and wastewater flowrates, minimization of the cost of regeneration and treatment costs) and with different methodologies (i.e., algorithmic, graphical, algebraic and mathematical programming approaches). In this paper the concept of eco-industrial parks refers to the inter-plant water integration.

Regarding water integration strategies in single plants, the minimization of the freshwater and/or wastewater flowrates have been addressed in several works through pinch analysis, such as those by Wang and Smith (1994), Kuo and Smith (1998a), Hallale (2002), El-Halwagi, Gabriel, and Harell (2003), Manan, Tan, and Foo (2004), Foo, Manan, and Tan (2006), Almutaq and El-Halwagi (2007), and Shenoy and Bandyopadhyay (2007). Bandyopadhyay and Cormos (2008) also used a graphical representation to address water management issues of integrated processes that involve regeneration and recycle through a single treatment unit. In addition, Kuo and Smith (1998b), Bandyopadhyay, Ghanekar and Pillai (2006), Agrawal and Shenoy (2006), Ng, Foo and Tan (2007a, 2007b), Ng, Foo, Tan and Tan (2007), Bai, Feng and Deng (2007) and Feng, Bai and Zheng (2007) have proposed targeting approaches for the minimization of regeneration costs and treatment flowrates. Recently, Ng, Foo and Tan (2009a, 2009b) proposed a linear model to determine the minimum resource consumption for single-impurity resource conservation networks, including an extension to determine the targets for resource conservation networks with interceptors. Also, several papers for mass integration based on properties as opposed to concentrations using the pinch analysis technique have been reported; such works have used graphical approaches (Bandyopadhyay, 2006; Kazantzi and El-Halwagi, 2005; Foo, Kazantzi, El-Halwagi and Manan, 2006) or algebraic/numerical methods (Foo, Kazantzi, El-Halwagi and Manan, 2006; Ng, Foo, Tan, Pau and Tan, 2009; Qin, Gabriel, Harell and El-Halwagi, 2004; Ng, Foo, Kazantzi and El-Halwagi,2006; Ng, Foo, Tan and El-Halwagi, 2010).

Introducing a mathematical optimization approach for single-plant integration, Takama,

On the works dealing with inter-plant integration using flowrate targeting techniques, Olesen and Polley (1996) presented one of the first methods based on pinch analysis. Spriggs, Lowe, Watz, El-Halwagi and Lovelady (2004) used the material recovery pinch diagram (El-Halwagi, Gabriel and Harell, 2003; Prakash and Shenoy, 2005) in inter-plant problems for fixed flowrates, but without detailing the targeting procedure. Foo (2008) addressed the targeting plant-wide integration using the numerical tool of water cascade analysis. This technique has also been employed for single water networks (Manan, Tan and Foo, 2004; Foo, Manan and Tan, 2006); however, this strategy is not recommendable for problems with large number of networks because prior to the calculation of minimum water flowrate targets, it is necessary to determine all alternative schemes for the inter-plant network. Recently, Bandyopadhyay, Sahu, Foo and Tan (2010) presented a generalized technique decomposition for determining optimal resource usage in segregated targeting problems with a single quality index through pinch analysis, and Chew, Foo and Ng (2010a, 2010b) presented a paper series based on pinch analysis for describing a new algorithm for targeting minimum fresh resource and waste flowrates for an inter-plant resource conservation network.

With respect to inter-plant integration using mathematical optimization, some papers have
been reported that allow the treatment of more complicated problems. Liao, Wu, Jiang, Wang, and Yang (2007) addressed the multi-period problem in multiple plant water networks with an approach applied for fixed contaminant operations and fixed flowrate operation, but limited to single contaminant problems. Lovelady, El-Halwagi, and Krishnagopalan (2007) reported a systematic approach for the reduction of water usage and wastewater discharge in pulp and paper plants; the model included mass integration strategies to handle multiple pollutants. Chew, Tan, Ng, Foo, Majozi and Gouws (2008) proposed an MINLP formulation for the synthesis of direct and indirect inter-plant water networks. Some limitations of this work are that the selection of the type of treatment unit was not set as an optimization variable, it did not allow direct flow rates between plants and the waste discharged to the environment, and it did not consider a limit for the pollutants concentration discharged to the environment. Lovelady, El-Halwagi, Chew, Ng, Foo and Tan (2009) developed a property-integration optimization approach for designing eco-industrial parks that are constrained by properties. Chew and Foo (2009) presented an automated targeting technique concept of pinch analysis combined with a mathematical optimization framework to locate the minimum flowrates/costs targets prior to detailed network design. Lovelady and El-Halwagi (2009) proposed a mass integration framework and a mathematical formulation for the design of eco-industrial parks for water integration. Lim and Park (2010) presented a nonlinear programming model to retrofit a conventional industrial park into a green eco-industrial park through the minimization of the total consumption of industrial water. The waste discharged to the environment was characterized by its flowrate, and the model did not consider environmental constraints for waste streams, cross-plant pipeline costs or regeneration costs. Also, the NLP model by Lim and Park (2010) is non convex, and the solution by standard optimization methods cannot guarantee a global optimal solution. Chen, Hung and Lee (2010) presented an MINLP problem for the inter-plant water integration of an industrial complex exploiting the opportunities for water reuse/recycle across plants. The model formulation was based on a superstructure and the synthesis task involved the optimal selection of treatment units. The model also considered the existence of multiple contaminants and limits for the flowrate and concentration for the contaminants discharged to the environment, although its solution cannot guarantee a global optimal solution. Aviso, Tan, Culaba and Cruz (2010) have developed a bi-level fuzzy optimization model to explore the effect of charging fees for the purchase of fresh water and treatment of wastewater in optimizing the water exchange network of plants in an eco-industrial park. An alternative fuzzy mathematical programming model to identify the optimal network that satisfies the fuzzy goals of the participating plants on eco-industrial parks has been reported by Aviso, Tan and Culaba (2010).
Chew, Tan, Foo, and Chiu, 2009; Chew, Thillaivarrna, Tan and Foo, 2010) have used game theory to analyze, model and design eco-industrial parks.

In this work, an approach for water integration in eco-industrial parks is presented. The approach takes into account multiple contaminants through an MINLP formulation, which is discretized to yield a mixed integer linear programming (MILP) problem so that a global or near global optimal solution can be obtained. Input data are specifications or limits for the flowrates and pollutants concentrations for a set of process sources and process sinks, the specific conversion factor for the interceptors considered to treat the sources, the pollutants concentration for each type of fresh water available, and the limits for discharges given by environmental regulations. The objective function is to minimize the total annual cost, which includes the cost of fresh water, treatment and cross-plant pipelines.

2. Model formulation

The proposed model is based on the superstructure of Figure 1, which shows two plants with two sources and two process sinks, two pollutants and one type of fresh water available. There is available an interception system for the eco-industrial park, and the treatment system is divided into stages. In each stage a set of interceptors with given efficiencies to remove specific pollutants is available. In this representation, each source can be segregated and directed to the interceptors of the first treatment stage, to the process sinks, and/or discharged to the environment; the exit flowrate from each interceptor in each treatment stage can be split and directed to any interceptor of the next treatment stage, and finally the flowrate from the interceptors of the last treatment stage can be divided and sent to any process sink and/or discharged to the environment. In addition, a fictitious interceptor is used for modeling the bypass stream when no treatment is required. In the model formulation, the subscript \( i \) is used to denote the process sources, \( j \) denotes the process sinks, \( t \) is used for the treatment stages, \( r_t \) is used to denote the type of interceptors used in stage \( t \), \( w \) is used to indicate the type of fresh water, and \( l \) denotes the pollutants. Superscripts \( \text{in}, \text{out}, \text{m} \) and \( \text{max} \) are used to denote inlet, outlet, removed mass and upper limit, respectively. \( NT \) is a scalar that represents the last stage of treatment units. For the sets, \( R_t \) is used to denote the interceptors considered in the treatment stage \( t \), \( J \) is the number of process sinks, \( I \) is used for the process sources, and \( T \) refers to the treatment stages. Notice that in each treatment stage it is only possible to treat a single contaminant; therefore, the number of treatment stages is equal to the number of pollutants to be removed. In addition, no special index for each plant is used; instead, the process sources and process sinks are enumerated consecutively to yield a convenient formulation. It should
be noted that a simplified superstructure for these types of problems has been reported by Chew, Tan, Ng, Foo, Majozi and Gouws (2008). The superstructure we present in this work (Figure 1) includes additional possible configurations such as the direct flowrates between plants, and the set of interceptors that are available in each treatment stage to remove the pollutant; also, the screening of multiple treatment processes and the selection of the optimal treatment process are taken here as decision variables.

The constraints for the model are as follows.

Mass balance for each process source

The flowrate of each process source \( FS_i \) can be divided and sent to the interceptors of the first treatment stage \( fsi_{r_{NT},i} \), to the process sinks in the same or different plants \( fss_{i,j} \), and/or discharged to the environment \( fse_i \).

\[
FS_i = \sum_{j=1}^{J} fss_{i,j} + \sum_{r_{NT}=1}^{R_{NT}} fsi_{r_{NT},i} + fse_i \quad i \in I \tag{1}
\]

Mass balance for each sink

The conditions in the inlet of any process sink regarding flowrate \( FU_j \) and pollutant concentrations \( cu_{j,l} \) are determined from the fractions of the flowrates of process sources \( fss_{i,j} \), fresh water \( fws_{w,j} \), and interceptors of the last treatment stage \( fis_{r_{NT},j} \) that are sent to each sink. The variables that appear in the component mass balance for the concentration and flowrate inlet to any sink are \( cl_{r_{NT},j}^{out} \) and \( fis_{r_{NT},j} \), and their products yield bilinear terms. Therefore, equation (3) for the component balance in the mixer prior to each sink is nonlinear and nonconvex,

\[
FU_j = \sum_{i=1}^{I} fss_{i,j} + \sum_{r_{NT}=1}^{R_{NT}} fis_{r_{NT},j} + \sum_{w=1}^{W} fws_{w,j} \quad j \in J \tag{2}
\]

\[
\sum_{i=1}^{I} cs_{i,l} fss_{i,j} + \sum_{r_{NT}=1}^{R_{NT}} cl_{r_{NT},j}^{out} fis_{r_{NT},j} + \sum_{w=1}^{W} cw_{w,l} fws_{w,j} \leq cu_{j,l} FU_j \quad j \in J; l \in L \tag{3}
\]

Here, \( cs_{i,l} \) is the concentration of pollutants in the process sources, \( cl_{r_{NT},j}^{out} \) is the outlet pollutants concentration in the interceptors of the last treatment stage, and \( cw_{w,l} \) is the pollutants concentration
in freshwater $w$.

**Mass balance in the interceptors of the first treatment stage**

The inlet flowrate to the interceptors considered in the first stage to remove pollutant one is equal to the fractions of flowrate of process sources ($f_{si,tir}$) directed to the interceptors.

$$FI_{r_i} = \sum_{j=1}^{L} f_{si,tir} \quad r_{i=1} \in R_{i=1}$$  \hspace{1cm} (4)

where $FI_{r_i}$ is the flowrate inlet to the interceptor $r_i$.

A component balance in the inlet of each interceptor is necessary to estimate the pollutants concentrations ($c_{tir}^{in}$). Equation (5) is nonlinear because the inlet flowrate and the pollutants concentrations are unknown.

$$c_{tir}^{in} = \sum_{i=1}^{L} c_{si,tir} f_{si,tir} \quad l \in L; r_{i=1} \in R_{i=1}$$  \hspace{1cm} (5)

Here, $c_{tir}^{in}$ is the inlet pollutants concentration in the interceptors.

**Mass balance in the inlet to interceptors after stage 1**

The flowrate inlet to the interceptors of stage $t \neq 1$ ($FI_{r_i}$) is the sum of the flows from the interceptors of treatment stage $t-1$ directed to each interceptor of the treatment stage $t$.

$$FI_{r_i} = \sum_{i=1}^{R} f_{ii,tir} \quad r_{i=1} \in R_{i=1}$$  \hspace{1cm} (6)

where $f_{ii,tir}$ is the flowrate from interceptor $r_{i-1}$ directed to the interceptor $r_i$ in the treatment stages after the first treatment stage.

To calculate the pollutant concentration inlet to the interceptors after stage 1, $c_{tir}^{in}$, the following component balance is used.

$$c_{tir}^{in} = \sum_{i=1}^{R} c_{tir}^{out} f_{ii,tir} \quad r_{i=1} \in R_{i=1}; l \in L$$  \hspace{1cm} (7)
where \( c_{r,t}^{\text{out}} \) is the outlet pollutant concentration in the interceptor \( r \) in stage \( t-1 \).

Notice that \( c_{r,t}^{\text{in}}, \quad FI_{r}^{t}, \quad c_{r,t}^{\text{out}}, \quad fi_{r,t}^{\text{r}} \) are unknown variables; therefore, equation (7) is nonlinear. For modeling reasons, this paper does not consider recycling streams between interceptors of the same treatment stage; this option would increase the number of bilinear terms, thus affecting the computational load for the problem solution.

Flowrates distribution in the interceptors

To meet the mass balance in the regeneration zone, the flowrate from the interceptors (\( FI_{r}^{t} \)) in stage \( t \) has to be equal to the flowrate distribution for the interceptors (\( fi_{r,t}^{\text{r}} \)) of stage \( t+1 \).

\[
FI_{r}^{t} = \sum_{r_{t+1}} \bar{fi}_{r_{t+1}} \quad r_{t+1} \in R_{r+NT} \tag{8}
\]

Notice that the above mass balance is only used from stage 1 to stage \( NT-1 \), because the flowrate of the last stage \( t = NT \) is split and directed to any process sink (\( fi_{s,t}^{\text{r}} \)) and/or to the environment (\( fie_{s,t}^{\text{r}} \)) as follows.

\[
FI_{r}^{t} = \sum_{j=1}^{N} fi_{s,t}^{r} + fie_{r}^{t} \quad r_{NT} \in R_{r+NT} \tag{9}
\]

Note that flowrates are not allowed from interceptors of the treatment stages before the last one to the sinks, but the existence of fictitious interceptors in each stage allow the streams to be sent to the sinks without further treatment. The above consideration was made to avoid additional segments of pipes to be built between different plants and the regeneration zone, thus preventing operational complications.

Interceptor balances

Each interceptor considered in the model has a given conversion factor (\( RR_{r,t}^{l} \)) for a specific pollutant, which is used to calculate the outlet pollutant concentration (\( c_{r,t}^{\text{out}} \)) from each interceptor in any treatment stage.

\[
c_{r,t}^{\text{out}} = c_{r,t}^{\text{in}} \left( 1 - RR_{r,t}^{l} \right) \quad r_{t} \in R_{t}, t \in T, l \in L \tag{10}
\]
The difference between the inlet and outlet concentration is the removed pollutant in each interceptor, which when multiplied by the flowrate provides the contaminant load \( (cim_{i,j}) \) that is used to determine the operating cost of the interceptor.

\[
cim_{i,j} = (ci_{i,j}^{in} - ci_{i,j}^{out})FI_{i,j} \quad r_i \in R_i; t \in T; l \in L
\]

One can notice that since \( ci_{i,j}^{in}, ci_{i,j}^{out} \) and \( FI_{i,j} \) are variables, equation (11) is nonlinear.

**Mass balance in the mixer prior to the waste discharged to the environment**

The flow rates can be segregated and sent to the waste stream discharged to the environment \((fse_i, fie_{e,ref})\) from each process sources and the last treatment stage of each interceptor. This determines the wastewater pollutant concentration \((ce_i)\) and flow rate \((FE)\) discharged to the environment as follows,

\[
FE = \sum_{i=1}^{l} fse_i + \sum_{t=ref}^{n} fie_{i,e}
\]

\[
ce_i FE = \sum_{i=1}^{l} cs_{i,j} fse_i + \sum_{t=ref}^{n} ci_{i,j}^{out} fie_{i,e} \quad l \in L
\]

The component mass balance (equation 13) is a nonlinear expression because the wastewater pollutants concentrations and flow rates are optimization variables.

**Determination of pipes**

In the following equations, \( x_{i,r}^1, x_{i,r}^2, x_{r,j}^3, \) and \( x_{r}^4 \) are binary variables used to determine the existence of pipes between process sources and sinks, process sources and interceptors, interceptors and sinks, and interceptors and waste discharged to the environment, respectively. The existence of the above pipes is determined using the following mixed integer formulations:

\[
fssi_{i,j} - M_{fssi_{i,j}}^{max} x_{i,j}^1 \leq 0 \quad i \in I; j \in J
\]

\[
fsti_{i,r \tau} - M_{fsti_{i,r \tau}}^{max} x_{i,r \tau}^2 \leq 0 \quad i \in I; \tau_{i=1} \in R_{\tau=1}
\]
\[ f_{s_{r_{NT}}} = m_{s_{r_{NT}}}^{\text{max}} x_{r_{NT}}^{3} \leq 0 \quad j \in J; r_{i} = NT \in R_{r_{i}} = NT \] 

\[ f_{e_{r_{NT}}} = m_{e_{r_{NT}}}^{\text{max}} x_{r_{NT}}^{4} \leq 0 \quad r_{i} = NT \in R_{r_{i}} = NT \] 

In equations (14-17) \( m_{f_{s_{i,j}}}^{\text{max}}, m_{f_{s_{i,1}}}^{\text{max}}, m_{f_{i,j}}^{\text{max}}, \) and \( m_{f_{e_{i}}}^{\text{max}} \) are the corresponding upper limits for each case.

**Feasibility for the flows**

Note that in the superstructure there are no flows between interceptors of the same treatment stage, since the sources can only be sent to the interceptors of the first treatment stage. Also, it is not possible to send the flowrate from any interceptor different than the ones of the last stage to the process sinks or to the environment, and the recirculation in the regeneration zone is not an option. Therefore, it is necessary to specify the elimination of these flowrates.

\[ f_{ii_{r_{i}}} = 0 \quad r_{i} \in R_{r_{i}}; \quad t \in T \] 

\[ f_{si_{i,r_{i}}} = 0 \quad i \in I; \quad r_{i} \in R_{r_{i}} ; \quad i \neq 1 \] 

\[ f_{is_{r_{i},j}} = 0 \quad j \in J; \quad r_{i} \neq NT \in R_{r_{i}} \neq NT \] 

\[ f_{ie_{r_{i}}} = 0 \quad r_{i} \neq NT \in R_{r_{i}} \neq NT \]

The above flowrates are eliminated to avoid additional complications within the problem formulation. It is worth noting here that the bypass streams are modeled through fictitious interceptors for each stage.

**Objective function**

The objective function consists of the minimization the total annual cost, \( TAC \), which includes the fresh water cost, \( WC \), regeneration cost, \( RC \), and the piping cost, \( PC \).

\[ TAC = WC + RC + PC \] 

The fresh water cost is calculated using the following relationship,
\[ WC = H_Y \sum_{w=1}^{W} \sum_{j=1}^{J} CUW_w f_{w,s_{w,j}} \] (23)

where \( H_Y \) represents the plant operating hours per year, and \( CUW_w \) is the unit cost of the fresh water \( w \).

The regeneration cost includes the fixed and operating costs for the interceptors.

\[ RC = \sum_{t_r=1}^{R} x_{t_r}^5 \text{CUI}_{t_r} + H_Y \sum_{t_r=1}^{R} \text{CUM}_{t_r} \text{cin}_{t_r,l} \quad t \in T; l \in L \] (24)

where \( \text{CUI}_{t_r} \) is a fixed unit cost and \( \text{CUM}_{t_r} \) is the unit cost for the mass removed in each interceptor. The binary variable \( x_{t_r}^5 \) is used to account for the fixed cost of interceptors. To activate this binary variable, the following relationship is used,

\[ FL_{t_r} - M_{FL_{t_r}} x_{t_r}^5 \leq 0 \] (25)

where \( M_{FL_{t_r}} \) is an upper limit for the allowable mass flowrate for any interceptor.

The piping cost includes fixed and operational costs for all pipe segments required for the mass integration network, and is calculated by (Kim and Smith, 2004; Chew, Tan, Ng, Foo, Majozi, and Gouws, 2008),

\[
PC = K_F \left\{ p \sum_{i=1}^{R} \sum_{j=1}^{J} \frac{D_{i,j} f_{s_{i,j}}}{3600 \ \rho v} + x_{i,j}^1 D_{i,j} \text{CUP}_p + p \sum_{i=1}^{R} \sum_{t_{r,s}=1}^{R_{r,s}} \frac{D_{i,t_{r,s}} f_{s_{i,t_{r,s}}}}{3600 \ \rho v} + x_{i,t_{r,s}}^2 D_{i,t_{r,s}} \text{CUP}_p + 
  p \sum_{i=1}^{R} \sum_{j=1}^{J} \frac{D_{i,j} f_{s_{i,j}}}{3600 \ \rho v} + x_{i,j}^3 D_{i,j} \text{CUP}_p + p \sum_{i=1}^{R} \sum_{t_{r,s}=1}^{R_{r,s}} \frac{D_{i,t_{r,s}} f_{s_{i,t_{r,s}}}}{3600 \ \rho v} + x_{i,t_{r,s}}^4 D_{i,t_{r,s}} \text{CUP}_p \right\} \] (26)

where \( K_F \) is an annualization factor, \( D \) is the length of the pipe segments, \( \rho \) is the water density, \( v \) is the velocity, \( p \) is a parameter for cross-plant pipeline cost, and \( \text{CUP}_p \) is the unit cost.

### 3. Bilinear terms reformulation

The original model is an MINLP problem because of the bilinear terms that appear in equations (3), (5), (7) and (11). To reformulate the model as a convex MILP problem, the bilinear terms need to be treated through a discretization approach. Note that the bilinear terms are
generated by the component balances in the mixers prior to the process sinks, interceptors, and waste discharged to the environment; however, the component balances in the process sinks and the waste discharged to the environment depend on the outlet conditions of the regeneration zone. In addition, the limits for the pollutant concentrations are given by the streams data. Then, from the data of the sources, it is possible to determine lower and upper limits for the concentration of the pollutants considered \( (C_{i}^{\text{min}}; C_{i}^{\text{max}}) \). When this range is discretized into known values, the bilinear terms become a convex linear problem, which allows getting near global optimal solutions. The range of the pollutant concentration is divided into \( n_q \) equal intervals. In such case, the values for the discretized pollutant concentrations \( l \) are calculated through the following equation (see Pham, Laird and El-Halwagi, 2009):

\[
C_{q,l} = C_{l}^{\text{min}} + \left( q - 1 \right) \frac{C_{l}^{\text{max}} - C_{l}^{\text{min}}}{n_q} \quad Q; \quad q = 1, \ldots, n_q + 1
\]  

(27)

where \( n_q \) is the number of splitting interval and \( q \) is an integer index associated to a discretized value.

Therefore, this work proposes the transformation of the original structure (Figure 1) into a discretized superstructure, where the concentration for the treatment units and the waste discharged to the environment are transformed into discretized units with known pollutants concentrations. Figure 2 shows the discretized superstructure for two plants including two process sources and two process sinks each, two pollutants, and one type of fresh water; two discretized interceptors for each specific conversion in each treatment stage and two discretized concentration for the waste discharged to the environment are considered. The splitting and mixing possibilities of the original superstructure are the same as in the discretized superstructure; in the latter, only the number of discrete possibilities is increased.

In the next section the discretized equations are presented.

### 3.1 Discretized model

The equations for the discretized model have the same physical explanation as in the original model. In the discretized equations, the subscript \( q_i \) is used to denote the discretized concentration for the interceptor in each stage, and \( q_e \) is used for the discretized concentration for the waste stream discharged to the environment. Superscripts min and max are used for lower and
upper limits. The sets $Q_i$ and $Q_e$ are used to denote the number of discretized terms for the treatment units and the waste discharged to the environment, respectively.

The reformulated discretized model is presented as follows.

**Mass balance for each process source**

$$FS_i = \sum_{j=1}^{I} fss_{i,j} + \sum_{r_{s1}=1}^{R_{s1}} \sum_{q_{s1}=1}^{Q_{s1}} fsi_{r_{s1},q_{s1}} + fse_i \quad i \in I$$  \hspace{1cm} (28)

**Mass balance for each process sink**

$$FU_j = \sum_{i=1}^{I} fss_{i,j} + \sum_{r_{ST}=1}^{R_{ST}} \sum_{q_{ST}=1}^{Q_{ST}} fis_{r_{ST},q_{ST},j} + \sum_{w=1}^{W} fws_{w,j} \quad j \in J$$  \hspace{1cm} (29)

$$\sum_{i=1}^{I} cIS_{i,j}fss_{i,j} + \sum_{r_{o1}=1}^{R_{o1}} \sum_{q_{o1}=1}^{Q_{o1}} cout_{r_{o1},q_{o1},j} fis_{r_{o1},q_{o1},j} + \sum_{w=1}^{W} cw_{j}fws_{w,j} \leq cu_{j}FU_{j} \quad j \in J; l \in L$$  \hspace{1cm} (30)

**Mass balance in the discretized interceptors of the first treatment stage**

$$FI_{r_{s1},q_{s1}} = \sum_{i=1}^{I} fis_{i,r_{s1},q_{s1}} \quad r_{s1} \in R_{s1}; q_{s1} \in Q_{s1}$$  \hspace{1cm} (31)

$$ci_{r_{s1},q_{s1}} \cdot FI_{r_{s1},q_{s1}} = \sum_{i=1}^{I} cs_{i,r_{s1},q_{s1}} fis_{i,r_{s1},q_{s1}} \quad r_{s1} \in R_{s1}; q_{s1} \in Q_{s1}; l \in L$$  \hspace{1cm} (32)

**Mass balance for the discretized interceptors in stage t+1**

$$FI_{r_{s1},q_{s1}} = \sum_{i=1}^{I} fis_{i,r_{s1},q_{s1}} \quad r_{s1} \in R_{s1}; q_{s1} \in Q_{s1}$$  \hspace{1cm} (33)

$$ci_{r_{s1},q_{s1}} \cdot FI_{r_{s1},q_{s1}} = \sum_{i=1}^{I} cs_{i,r_{s1},q_{s1}} fis_{i,r_{s1},q_{s1}} \quad r_{s1} \in R_{s1}; q_{s1} \in Q_{s1}; l \in L$$  \hspace{1cm} (34)

**Discretized flowrates distribution for interceptors**

$$FI_{r_{s1},q_{s1}} = \sum_{i=1}^{I} fis_{i,r_{s1},q_{s1}} \quad r_{s1} \in R_{s1}; q_{s1} \in Q_{s1}$$  \hspace{1cm} (35)
\begin{equation}
FI_{r_i,q_i} = \sum_{j=1}^{I} f_{i,j} r_i,q_i + \sum_{q_i=1}^{Q_i} f_{i,q_i} r_i,q_i \quad r_i \in R_{i} ; q_i \in Q_{i} ; t \in T ; l \in L
\end{equation}

Outlet concentration, contaminant load and inlet concentration for the pollutants in the discretized interceptors

\begin{equation}
c_{\text{in}}^{\text{out},r_i,q_i,j} = c_{\text{in}}^{\text{in},r_i,q_i,j} \left(1 - RR_{r_i,q_i,j}\right) \quad r_i \in R_{i} ; q_i \in Q_{i} ; t \in T ; l \in L
\end{equation}

\begin{equation}
c_{\text{in}}^{\text{in},r_i,q_i,j} = \left(c_{\text{in}}^{\text{in},r_i,q_i,j} - c_{\text{in}}^{\text{out},r_i,q_i,j}\right) FI_{r_i,q_i} \quad r_i \in R_{i} ; q_i \in Q_{i} ; t \in T ; l \in L
\end{equation}

Notice that when the inlet concentration in each discretized interceptor \( c_{\text{in}}^{\text{in},r_i,q_i,j} \) is specified, then the outlet concentration \( c_{\text{in}}^{\text{out},r_i,q_i,j} \) and the removed concentration \( c_{\text{in}}^{\text{out},r_i,q_i,j} \) of the pollutants are transformed into known values; therefore, all component balances become linear. The expression to obtain the discretized values for inlet pollutant concentration is as follows,

\begin{equation}
c_{\text{in}}^{\text{in},r_i,q_i,j} = c_{\text{in}}^{\text{in},\min,r_i,q_i,j} + \left(q_i - 1\right) \frac{c_{\text{in}}^{\text{in},\max,r_i,q_i,j} - c_{\text{in}}^{\text{in},\min,r_i,q_i,j}}{n_{q_i}} \quad q_i = 1, \ldots, n_{q_i} + 1
\end{equation}

Mass balance in the mixer prior to the waste discharged to the environment

\begin{equation}
FE_{q_e} = \sum_{i=1}^{I} f_{i} s_{i,q_e} + \sum_{r_i=1}^{R_{i}} \sum_{q_i=1}^{Q_{i}} f_{i,r_i,q_i,q_e} \quad q_e \in Q_{e}
\end{equation}

\begin{equation}
CE_{q_e,l} FE_{q_e} = \sum_{i=1}^{I} c_{i} s_{i,q_e} + \sum_{r_i=1}^{R_{i}} \sum_{q_i=1}^{Q_{i}} c_{i,r_i,q_i,l} f_{i} s_{i,q_e} \quad q_e \in Q_{e} ; l \in L
\end{equation}

To transform the above equation into a linear expression, it is required to specify the values for the pollutants concentration in the discretized waste stream discharged to the environment as follows,

\begin{equation}
CE_{q_e,l} = c_{\text{min}}^{\text{in},q_e,l} + \left(z_{q_e} - 1\right) \frac{c_{\text{max}}^{\text{in},q_e,l} - c_{\text{min}}^{\text{in},q_e,l}}{n_{q_e}} \quad q_e \in Q_{e} ; z_{q_e} = 1, \ldots, n_{q_e} + 1
\end{equation}

At most one discretized waste discharged to the environment must be selected, and to activate such discretized term the following relationship is used,
\[ FE_{q_e} - M_{FE_{q_e}}^{\max} x_e^6 \leq 0 \quad q_e \in Q_e \]  

(43)

\[ \sum_{q_e=1}^{Q_e} x_e^6 \leq 1 \quad t \in T \]  

(44)

where \( FE_{q_e} \) is the flowrate in the discretized waste discharged to the environment, \( M_{FE_{q_e}}^{\max} \) is an upper limit for the waste stream, and \( x_e^6 \) is a binary variable used to activate any discretized discharge to the environment.

**Determination of pipes**

The relationships used to determine the pipes for the segments process sources-process sinks are the same as the ones proposed in equation (14) because the process sinks are not discretized. However, because the interceptors and the waste discharged to the environment are discretized, the relationships for the segments process sources-interceptors, interceptors-process sinks and interceptors-waste discharged to the environment must be reformulated as follows:

\[ f_{si_{i,r,q}}^s - M_{fsi_{i,r,q}}^{\max} x_{i,r,q}^2 \leq 0 \quad i \in I; r_{t=1} \in R_{t=1}; q_{t=1} \in Q_{t=1} \]  

(45)

\[ f_{is_{r,q,j}}^s - M_{fis_{r,q,j}}^{\max} x_{r,q,j}^3 \leq 0 \quad j \in J; r_{t=NT} \in R_{t=NT}; q_{t=NT} \in Q_{t=NT} \]  

(46)

\[ f_{ie_{r,q,q_e}}^s - M_{fie_{r,q,q_e}}^{\max} x_{r,q,q_e}^4 \leq 0 \quad r_{t=NT} \in R_{t=NT}; q_{t=NT} \in Q_{t=NT}; q_e \in Q_e \]  

(47)

**Elimination of flows**

\[ f_{ii_{i,q_i,q_i,q_i}} = 0 \quad r_{i} \in R; \hat{\mu}_{i} \in Q; \hat{\nu} \in T \]  

(48)

\[ f_{si_{i,r,q}}^i = 0 \quad i \in I; \hat{\beta}_{i,r} \in R_{i,r}; \hat{\nu}_{i,r} \in Q_{i,r} \]  

(49)

\[ f_{is_{r,q,j}}^s = 0 \quad j \in J; \hat{\beta}_{r,q,j} \in R_{r,q,j}; \hat{\nu}_{r,q,j} \in Q_{r,q,j} \]  

(50)

\[ f_{ie_{r,q,q_e}} = 0 \quad r_{t=NT} \in R_{t=NT}; \hat{\beta}_{r,q,q_e} \in Q_{r,q,q_e} \]  

(51)

**Regeneration cost**
\[
RC = R \sum_{r=1}^{R} \sum_{q=1}^{Q} \chi_{r,q}^5 CUI_{r,q} + H \sum_{r=1}^{R} \sum_{q=1}^{Q} CUM_{r,q} cim_{r,q,l} \quad t \in T; l \in L
\]  

(52)

The binary variable \( \chi_{r,q}^5 \) is used to account for the fixed cost of the discretized interceptors.

To activate this binary variable, the following relationship is used,

\[
F_{I_{r,q}} - M_{I_{r,q}} \chi_{r,q}^5 \leq 0 \quad r \in R; t \in Q; l \in T
\]

(53)

Here, \( M_{I_{r,q}} \) is an upper limit for the allowable mass flowrate for any discretized interceptor. Because it is only possible to select at most one discretized interceptor with a specific conversion factor, the following constraint is required,

\[
\sum_{q=1}^{Q} \chi_{r,q}^5 \leq 1 \quad t \in T
\]

(54)

Model Remarks

- Equations (1)-(26) constitute the original nonconvex MINLP model, for which the global optimal solution cannot be guaranteed with conventional techniques (for example the outer approximation method by Viswanathan and Grossmann, 1990). In addition, the possibility to find a local feasible solution depends on good initials estimates and good limits for the major variables.

- The linear MILP reformulated model given by equations (14), (22), (23), (26) and (28)-(54) is a convex problem, whose solution yields a global or near global optimal solution. In the MILP problem, it is not necessary to determine limits or initial values for the optimization variables.

- The size of the problem depends on the number of pollutants and number of split intervals. For instance, consider two pollutants and 21 intervals to split the range of the pollutants concentration, as shown schematically in Figure 3. Then, each discretized concentration of pollutant 1 can be combined with every discretized vale of the pollutant 2, thus generating 441 possibilities.

- The discretization strategy used in this work overcomes the problems of other global optimization techniques (e.g. Quesada and Grossmann, 1995), which show difficulties to provide good lower bounds for problems with several bilinear terms (see Ruiz and
Grossmann, 2010) and that depend on initial guesses to determine proper upper bounds.

4. Case Studies

Four examples are considered to show the application of the proposed model. Each problem was solved using both inter-plant integration and single plant integration with the CPLEX solver included in the GAMS software (Brooke, Kendrick and Meeraus, 2006). For all examples, the parameters $D$, $K_F$, $H_Y$, $CUI$, $CUP$, $v$ and $\rho$ were 100 m, 0.231/year, 8000 hr/year, 12600 US$, 250 US$, 1 m/s and 1000 kg/m$^3$, respectively. In addition, two interceptors for Examples 1, 2 and 3 and eight interceptors for Example 4 were considered with the conversion factor ($RR_e$) and unit cost for mass removed ($CUM_e$) given in Table 1 (El-Halwagi, 2006).

Example 1. This example consists of three plants with two process sources and two process sinks each one, with the design data given in Table 2. Two pollutants are considered for the integration task, and clean fresh water with a unit cost of 0.13 US$/ton is available. The limit for the concentration of the stream discharged to the environment for both contaminants is 85 ppm. The minimum and maximum values for the concentration of pollutant 1 in the inlet regeneration zone are 80 and 125 ppm, obtained by the inspection of the streams data; the corresponding values for the concentration of pollutant 2 are 70 and 120 ppm.

Figure 4 presents the solution obtained for the case when the plants are integrated separately (whose solution was also obtained through the application of the discretization approach). The solution of this problem allowing inter-plant integration yields the configuration shown in Figure 5. The MILP problem consisted of 1,072 binary variables, 4,487 continuous variables and 1,520 constraints. Note that in addition to the integration in the same plant, different industries exchange streams in the optimal solution. The interceptor 1 is used to treat component 2 in the second treatment stage, whereas the treatment of pollutant 1 is not needed. It is worth noticing that plant 2 and plant 3 are integrated through the flowrate of the source 3 (plant 2) to the sink 6 (plant 3), whereas part of the sources 3 and 4 (plant 2) and sources 5 and 6 (plant 3) are split and sent directly to the environment. The major difference between the solutions of Figures 4 and 5 is that the no inter-plant integration solution uses two interceptors, one in plant 1 and the other one in plant 2. Table 3 presents a comparison of results, from which one can notice that the inter-plant integration solution is 33.9% cheaper than the single plant integration solution, basically because of lower costs.
in fresh water, regeneration and cross-plant pipeline. In addition, the inter-plant solution uses 23.4% less fresh water (i.e., discharges 23.4% less waste to the environment).

**Example 2.** This example includes three plants with five process sources and five process sinks; flowrates and pollutant concentrations are given in Table 4. Two pollutants are considered, and two fresh sources are available, fresh water without pollutant with a unit cost of 0.13 US$/ton, and fresh water with a cost of 0.10 US$/ton with 5 ppm and 0.075 ppm of pollutants 1 and 2. The limits for the pollutant concentrations for the wastewater discharged to the environment are 60 ppm for pollutant 1 and 3 ppm for pollutant 2. First, Example 2 was solved without considering the inter-plant integration, and the solution is shown in Figure 6. Notice that two interceptors for plant 1 and one for plant 3 are required. Three plants discharge wastewater to the environment. The reformulated MILP problem for the inter-plant integration consists of 2,470 binary variables, 7,665 continuous variables and 2,970 constraints, and the solution of this problem yields the configuration shown in Figure 7. In this case, it was only necessary the treatment of pollutant 2 using interceptor two in the second treatment stage (in the first treatment stage the fictitious interceptor that only mixes the streams was selected). The inlet flowrate to the regeneration zone is constituted by sources from the three plants, and the outlet flowrate from the regeneration zone is split and sent to plants 1 and 2 to meet the constraints in the process sinks. In addition, the use of fresh water with pollutants (417 ton/hr) is higher than the use of the clean fresh water (75 ton/hr), and process sources from the three plants are discharged directly to the environment. The costs for the configurations shown in Figures 6 and 7 are presented in Table 5. The inter-plant integration allows significant savings in the regeneration and cross-plant pipeline costs, which are translated into a reduction of 5.8% of the total annual cost.

**Example 3.** Table 6 shows the data for this example of three plants with five process sources and five process sinks each one. Three pollutants and three types of fresh water are considered. The unit cost for each type of fresh water is 0.13 US$/ton, 0.11 US$/ton and 0.10 US$/ton. The costs of fresh water reflect the pollutant content; type 1 is clean water, type 2 contains 10 ppm, 15 ppm and 20 ppm of the pollutants 1, 2 and 3, and type 3 contains 5 ppm of pollutant 1, 10 ppm of pollutant 2 and no pollutant 3. From the streams data one can determine the upper limits for the waste stream discharged to the environment, which are 80 ppm, 100 ppm and 70 ppm for pollutants 1, 2 and 3.

Figure 8 shows the solution for this case considering only single plant integration. When inter-plant integration was considered, the MILP problem consisted of 14,771 constraints, 97,723
continuous variables and 11,697 binary variables. The optimal configuration obtained is shown in Figure 9. One can see that there are several inter-plant streams in this structure, and that the interceptors of type 1 are selected to treat the pollutants 2 and 3. A comparison of results for the solutions with and without interplant integration is given in Table 7, from which one can see that savings of 29% in the total annual cost are obtained when inter-plant integration was allowed.

Example 4. This example was previously addressed by Chew, Tan, Ng, Foo, Majozi and Gouws (2008), and originally reported by Oelsen and Polley (1996). This is a single-contaminant problem that is taken here to compare the application of the proposed model to a reported solution in the literature, and to show the advantages provided by the present formulation (i.e., to consider the optimal selection of a set of interceptors as well as direct flowrates between plants). To allow for a consistent comparison, no limit was imposed on the waste discharged to the environment. The sources and sinks data are shown in Table 8. The MILP model for the application of the proposed strategy consists of 8,597 constraints, 10,649 continuous variables and 4,134 binary variables. The optimal configuration provided by the model of this work is shown in Figure 10. Some differences with respect to the reference work can be detected. Such differences include the selection of two interceptors (it should be noted that in the method of the reference work the selection of interceptors is not optimized simultaneously and only one interceptor can be used), and the direct flowrate that is observed from plant 2 to plant 1 (this option was not allowed in the model of the reference work). Overall, the structure provided by the model of this work yields savings in the total annual cost and the fresh water consumption of 5.6% and 3.4% with respect to the solution obtained in the reference work. The major differences arise in the regeneration and pipeline costs, as can be seen in Table 9. The CPU time required for the new solution is higher than the one taken to solve the problem with the model of the reference work because of the larger number of possibilities here considered and the implementation of the global optimization approach.

Finally, we notice that the computational time required for the solution of the above examples was not a major issue, as can be seen in Tables 3, 5, 7 and 9.

5. Conclusions

A new strategy for water integration in eco-industrial parks considering multiple pollutants has been proposed. The model extends the superstructure reported by Chew, Tan, Ng, Foo, Majozi and Gouws (2008) to include additional network configurations, from which the optimal one should be selected; in the superstructure, the process sources can be segregated and directed to the regeneration zone, to process sinks, or be discharged to the environment. The flowrate sent to the
regeneration zone, after it is treated, can be segregated and sent to the process sinks and/or discharged to the environment. The selection of each pollutant interceptor is optimized simultaneously. The model considers environmental constraints for the concentration of the pollutants discharged to the environment. The original model gives rise to an MINLP problem that contains several bilinear terms. The MINLP problem is then reformulated as an MILP convex problem that can be used to find a global or near global optimal solution; the MILP problem does not require limits and initial values for the optimization variables for its solution. The application to four case studies has shown that there are important economic incentives to consider water inter-plant integration policies as opposed to single plant integration strategies.

**Nomenclature**

- \( ce_i \): concentration of pollutant \( l \) in the waste stream discharged to the environment, ppm
- \( ce_{q,l} \): concentration of pollutant \( l \) in the discretized stream discharged to the environment \( q_e \), ppm
- \( cim_{r,q,l} \): load of pollutant \( l \) in the interceptor \( r \) in treatment stage \( t \), ppm
- \( cim_{r,q,l}^{in} \): load of pollutant \( l \) in the discretized interceptor \( r, q_l \) in treatment stage \( t \), ppm
- \( cim_{r,q,l}^{in} \): inlet concentration of pollutant \( l \) in the discretized interceptor \( r, q_l \) in treatment stage \( t \), ppm
- \( cim_{r,q,l}^{out} \): outlet concentration of pollutant \( l \) from the interceptor \( r \) in treatment stage \( t \), ppm
- \( cim_{r,q,l}^{out} \): outlet concentration of pollutant \( l \) in the discretized interceptor \( r, q_l \) from treatment stage \( t \), ppm
- \( C_{q_l}^{min} \): lower limit for the concentration of pollutant \( l \) in the discretized waste discharged to the environment \( q_e \), ppm
- \( C_{q_l}^{max} \): upper limit for the concentration of pollutant \( l \) in the discretized waste discharged to the environment \( q_e \), ppm
\( C_{i_{\min}} \) lower limit for the concentration of pollutant \( l \), ppm

\( c_{r_{\min}} \) lower limit for the inlet concentration of pollutant \( l \) to the discretized interceptor \( r_s q_t \), ppm

\( C_{i_{\max}} \) upper limit for the concentration of pollutant \( l \), ppm

\( c_{r_{\max}} \) upper limit for the inlet concentration of pollutant \( l \) in the discretized interceptor \( r_s q_t \), ppm

\( C_{q,l} \) discretized value \( q \) of the concentration of pollutant \( l \), ppm

\( c_{s_{i,l}} \) concentration of pollutant \( l \) in source \( i \), ppm

\( c_{u_{j,l}} \) concentration of pollutant \( l \) in process sink \( j \), ppm

\( CUI_{i} \) fixed unit cost of interceptor \( r_s \), US$

\( CUI_{r_{s}q_{t}} \) fixed unit cost of discretized interceptor \( r_s q_t \), US$

\( CUM_{r_{s}} \) unit cost for mass removed in interceptor \( r_s \), US$/kg

\( CUM_{r_{s}q_{t}} \) unit cost for mass removed in the discretized interceptor \( r_s q_t \), US$/kg

\( CUP_{p} \) unit cost for pipeline, US$

\( CUW_{w} \) unit cost of fresh water \( w \), US$/ton

\( c_{w_{l}} \) concentration of pollutant \( l \) in fresh water \( w \), ppm

\( D \) distance between sources and process sinks, m

\( FE \) flowrate for the waste discharge to the environment, ton/hr

\( FE_{q_{c}} \) flowrate in the discretized discharge to the environment \( q_{c} \), ton/hr

\( f_{ii_{r_{s}}-r_{t}} \) flowrate for interceptor \( r_i \) from treatment stage \( t-1 \) to interceptor \( r_t \) of treatment stage \( t \), ton/hr

\( f_{ii_{r_{s}}-q_{t}} \) flowrate of discretized interceptor \( r_s q_t \) of treatment stage \( t-1 \) to discretized
interceptor $r, q_t$ of treatment stage $t$, ton/hr

$fii_{r, r_{t-1}}$ flowrate of interceptor $r_t$ of treatment stage $t$ to interceptor $r_{t-1}$ of treatment stage $t-1$, ton/hr

$fii_{r, q_{r_{t-1}}, r_{t-1}, q_{t-1}}$ flowrate of discretized interceptor $r, q_{t-1}$ of treatment stage $t$ to the discretized interceptor $r_{t-1}, q_{t-1}$ of treatment stage $t-1$, ton/hr

$fie_{r, i}$ flowrate of interceptor $r_i$ in treatment stage $t=NT$ to the waste discharged to the environment, ton/hr

$fie_{r, q_{r_{t-1}}}$ flowrate of the discretized interceptor $r, q_{t-1}$ in treatment stage $t=NT$ to the discretized waste discharged to the environment $q_e$, ton/hr

$F I_{r, i}$ flowrate of the interceptor $r_i$ in treatment stage $t$, ton/hr

$F I_{r, q_{r_{t-1}}}$ flowrate of the discretized interceptor $r, q_{t-1}$ in treatment stage $t$, ton/hr

$fis_{r, j}$ flowrate from interceptor $r_i$ in stage treatment $t=NT$ to process sink $j$, ton/hr

$fis_{r, q_{r_{t-1}}, j}$ flowrate from discretized interceptor $r, q_{t-1}$ in treatment stage $t=NT$ to process sink $j$, ton/hr

$F S_{i, I}$ flowrate of process sources $i$, ton/hr

$fse_{i, e}$ flowrate from source $i$ to the waste discharged to the environment, ton/hr

$fse_{i, q_e}$ flowrate from source $i$ to the discretized waste discharged to the environment $q_e$, ton/hr

$fsi_{i, r_i}$ flowrate of source $i$ to interceptor $r_i$ in treatment stage $t=I$, ton/hr

$fsi_{i, q_{r_{t-1}}}$ flowrate of source $i$ to the discretized interceptor $r, q_{t-1}$ in treatment stage $t=I$, ton/hr

$fss_{i, j}$ flowrate from source $i$ to process sink $j$, ton/hr

$F U_{j, I}$ flowrate in process sink $j$, ton/hr

$fws_{w, j}$ flowrate of the type of fresh water $w$ in process sink $j$, ton/hr

$H_Y$ plant operating hours per year, hr/year
$K_F$  annualization factor, year$^{-1}$

$M_{FE}^{\text{max}}$  upper limit for the flowrate in the discretized waste discharged to the environment $q_e$

$M_{FI}^{\text{max}}$  upper limit for the flowrate for interceptor $r_i$ in treatment stage $t$

$M_{FI_{-\infty}}^{\text{max}}$  upper limit for the flowrate in the discretized interceptor $r_i,q_i$ in treatment stage $t$

$M_{fss_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of source $i$ to sink $j$

$M_{fsi_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of source $i$ to interceptor $r_{t=1}$

$M_{fss_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of source $i$ to discretized interceptor $r_{t=1}, q_{t=1}$

$M_{fix_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of interceptor $r_{t=NT}$ to sink $j$

$M_{fix_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of discretized interceptor $r_{t=NT}, q_{t=NT}$ to sink $j$

$M_{fix_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of interceptor $r_{t=NT}$ to waste discharged to environment

$M_{fie_{ij}}^{\text{max}}$  upper limit for the flowrate in pipe segment of discretized interceptor $r_{t=NT}, q_{t=NT}$ to waste discharged to environment

$n_q$  number of intervals for splitting the range of concentration

$n_{q}$  number of intervals for splitting the range of concentration in treatment stage $t$

$PC$  cross-plant pipeline cost, US$/year

$p$  parameter for cross-plant pipeline cost

$RC$  regeneration cost, US$/year
conversion factor for interceptor \( r \) in treatment stage \( t \), dimensionless

- **TAC** total annual cost, US$/year
- **\( v \)** velocity, m/s
- **WC** fresh water cost, US$/year

**Binary variables**

\( x_e^6 \) binary variable to determine the existence of the discretized waste discharged to the environment \( q_e \)

\( x_{i,j}^1 \) binary variable to determine the segment of pipe from source \( i \) to process sink \( j \)

\( x_{i,r}^2 \) binary variable to determine the segment of pipe from source \( i \) to interceptor \( r \) in the treatment stage \( t=1 \)

\( x_{i,r,q}^2 \) binary variable to determine the pipe segment from source \( i \) to discretized interceptor \( r,q \) in the treatment stage \( t=1 \)

\( x_{r}^5 \) binary variable used to account for the fixed cost for the interceptors \( r \) in the treatment stage \( t \)

\( x_{r,q}^5 \) binary variable used to account for the fixed cost for the discretized interceptors \( r,q \) in the treatment stage \( t \)

\( x_{r,j}^3 \) binary variable used to determine the pipe segment from interceptor \( r \) in the treatment stage \( t=NT \) to process sink \( j \)

\( x_{r,q,j}^3 \) binary variable used to determine the pipe segment from discretized interceptor \( r,q \) in the treatment stage \( t=NT \) to process sink \( j \)

\( x_{r}^4 \) binary variable used to determine the pipe segment from interceptor \( r \) in the treatment stage \( t=NT \) to the waste discharged to the environment
$x^4_{r_t,q_{r},q_{t}}$ binary variable used to determine the pipe segment for discretized interceptor $r_t,q_{r}$ in the treatment stage $t=NT$ to the discretized waste discharged to the environment $q_e$

**Greek symbols**

$\rho$ water density, kg/m$^3$

**Subscripts**

$i$ source

$j$ sink

$l$ pollutant

$q_{t_t}$ discretized interceptor in treatment stage $t$

$q_{e_t}$ discretized waste discharged to the environment

$r_t$ type of interceptor in the stage $t$

$t$ treatment stage

$w$ type of fresh water

**Superscripts**

$in$ inlet

$m$ removed mass

$max$ upper limit

$out$ outlet

** Scalars**

$NT$ last treatment stage

**Sets**

$I$ \{i=1,2,…,N$_{sources}$\} $I$ is a set of process sources}
\[
J = \{j=1,2, \ldots, N_{\text{sinks}}\} \text{ is a set of process sinks}
\]
\[
Q_t = \{q_t=1,2, \ldots, N_{\text{discretized interceptors}}\} \text{ } Q_t \text{ is a set of discretized interceptors in the treatment stage } t
\]
\[
Q_e = \{q_e=1,2, \ldots, N_{\text{discretized environment discharges}}\} \text{ } Q_e \text{ is a set of discretized environment discharges}
\]
\[
R_t = \{r_t=1,2, \ldots, N_{\text{interceptors}}\} \text{ } R_t \text{ is a set of interceptors in treatment stage } t
\]
\[
T = \{t=1,2, \ldots, N_{\text{treatment stages}}\} \text{ } T \text{ is a set of treatment stages}
\]

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<table>
<thead>
<tr>
<th>RR</th>
<th>CUM ($/kg-removed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Examples 1, 2 and 3</td>
</tr>
<tr>
<td>0.6</td>
<td>1.460</td>
</tr>
<tr>
<td>0.8</td>
<td>2.060</td>
</tr>
<tr>
<td></td>
<td>Example 4</td>
</tr>
<tr>
<td>0.1</td>
<td>0.540</td>
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<tr>
<td>0.2</td>
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<td>0.3</td>
<td>0.850</td>
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<td>1.005</td>
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<tr>
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<td>1.160</td>
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Table 2. Data for Example 1

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Table 5. Results for Example 2

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**Table 6.** Data for Example 3

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Table 7. Results for Example 3

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Table 9. Comparison for Example 4

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List of Figures

Figure 1. Superstructure for water integration in eco-industrial parks

Figure 2. Discretized superstructure for water integration in eco-industrial parks

Figure 3. Discretized values and possible combinations for two pollutants

Figure 4. Optimal configuration for Example 1 without inter-plant integration

Figure 5. Optimal configuration for Example 1 with inter-plant integration

Figure 6. Optimal configuration for Example 2 without inter-plant integration

Figure 7. Optimal configuration for Example 2 with inter-plant integration

Figure 8. Optimal configuration for Example 3 without inter-plant integration

Figure 9. Optimal configuration for Example 3 with inter-plant integration

Figure 10. Optimal configuration for Example 4 for inter-plant integration
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