Distributed Turbo Block Codes in Wireless Relay Networks

Abstract—This paper proposes distributed turbo block codes (DTBC) by using extended Bose, Chaudhuri, Hocquenghem (eBCH) codes as the constituent codes in a simple relay networks. All channels are considered to be orthogonal and two phases are required to complete the transmission. In the first phase, the source broadcasts the information sequence together with the parity sequence from the first code. The relay received the data from the source and process it by using decode and forward relay protocol. The decoded systematic data at relay is interleaved and encoded by the second encoder. In the second phase, the parity sequence generated at relay is forwarded to the destination. In order to retrieve the original information bits, the destination process the signals received from the source and relay by using turbo iterative decoding technique, and soft-input soft-output Chase-Phyndiah (CP) algorithm is used in each decoder component. We have conducted a series of simulations to study the effect of different iteration numbers at destination, different numbers of test pattern for Chase algorithm at destination and different locations of relay in a line network model. From our results, we have shown that the proposed cooperative scheme outperforms the non-cooperative system as long as the relay is not too far away from the source. Finally, we have presented the results of the proposed system by using different constituent codes with different minimum Hamming distances.

I. INTRODUCTION

In the past few years, multiple-input multiple-output (MIMO) techniques have become one of the most attractive solutions to combat fading in wireless environment as it can provide spatial diversity gain. However, the implementation of MIMO requires increase of size and complexity of the communication systems, hence involving high cost and not practical for implementation in some networks (i.e. wireless sensor networks). In order to avoid these problems, a relay system has been proposed [1], in which a relay or intermediate node is introduced between the source and destination. This intermediate node can be seen as the second antenna of the source and in some literatures, the relay system is known as virtual MIMO. The structure of the relay system enables the communications between the nodes are done in cooperative manner.

In cooperative communications, the signal processing at relay is the crucial part, where it determines the overall performance of the networks. In order to achieve diversity gain and coding gain, several coding schemes have been proposed in order to utilize the distributed nature in relay networks. Based on the decode and forward relay protocol, distributed turbo codes (DTC) have shown to achieve good performance, as it can provide interleaving gain on top of the coding and diversity gain [2]. Due to its outstanding performance, many research have been focusing on using DTC as the channel coding and convolutional codes have been used as the constituent codes. Meanwhile, parallel concatenated block codes or turbo block codes can achieve powerful BER performance with high code rate [3]–[5]. However, existing works on turbo block codes are only done for point to point communications and no work has been done to extend it in cooperative communications. Similar work based on block codes has been presented in [6] by using turbo product codes for multiple relays. The difference of work in [6] with our work is that we use parallel concatenated codes structure, in which redundancy part comes from the parity of parity check of the codes is omitted to enable higher code rate. Moreover, we use list-based algorithm for the decoding at destination while [6] uses distance-based algorithm.

In this work, a novel high rate distributed turbo block codes (DTBC) is proposed for a simple relay network consists of single source, single relay and single destination nodes where all nodes use single antenna. Iterative decoding based on soft-input soft-output Chase Phyndiah (CP) algorithm [7] is used at destination in order to retrieve the original information bits. We have investigated the performance of the proposed system in three terminals line network model where the relay is located in a line between the source and destination. The effects of relay location, codes with different minimum Hamming distance, and the number of Chase test pattern have been investigated through simulations.

The remainder of the paper is organized as follows. Brief description of the system model is described in section II. Sections III presents the simulation results of the proposed system by using CP algorithm for the turbo decoding at destination. Finally, concluding remarks are provided in Section IV.

II. SYSTEM DESCRIPTION

We have considered a simple relay system following the three terminal line network model as shown in Fig. 1 where \( L \) is the distance between source and destination and \( 0 \leq d \leq 1 \).
A. Decoding at relay

In the second phase, the relay process received signal $\mathbf{y}_{SR}$ from the source by using decode and forward strategy. The received signal is decoded by using soft-input hard-output Chase 2 algorithm [8]. Chase 2 is a list-based decoding algorithm where the decoded codeword are determined from a list of codeword candidates. The procedure in Chase 2 algorithm can be explained by the following steps:

Step 1: The position of the $p$ least reliable bit is determined from the reliability of the received signal, $L = \frac{2}{\sigma_{SD}^2}\mathbf{y}_{SR}$.

Step 2: $q (= 2^p)$ test patterns, $TP_q$ are formed that have any combination of 1s located in the $p$ least reliable positions and 0s in the other locations.

Step 3: Test sequences $Z^q$ are formed where $Z^q = TP_q \oplus L$. Then, $Z^q$ is decoded by using algebraic decoding. In this paper, we use Berlekamp algorithm [9] for the algebraic decoding. Successful decoded codewords are added into the candidate list.

Based from the candidate list of codewords, the decision $\hat{\mathbf{u}}_R$ is given by using maximum likelihood decoding (MLD) where the codeword with the least Euclidean distance from the received vector is chosen. Once $\hat{\mathbf{u}}_R$ has been determined, $\hat{\mathbf{u}}_R$ is interleaved by using block interleaver of size $k \times k$ and re-encoded by using the encoder $C_2$. Only the parity part of the codeword is taken and transmitted as signal $\mathbf{x}_R$ before forwarded to the destination. During this phase, the source is in idle mode and the received signal at destination from relay can be expressed as:

$$\mathbf{y}_{RD} = \mathbf{x}_R + \mathbf{z}_{RD},$$

where $\mathbf{z}_{RD}$ is the AWGN noise with zero mean and variance of $\sigma_{RD}^2$. The transmitted power from the source and relay is assumed equal and therefore $\sigma_{RD}^2$ and the SNR for relay-destination link, $SNR_{RD}$ can be expressed as:

$$\sigma_{RD}^2 = (1 - d)^2 \sigma_{SD}^2,$$

$$SNR_{RD} = \frac{SNR_{SD}}{(1 - d)^2}.\quad (7)$$

The overall code rate of the system is given by $k/(2n - k)$.

The destination receives two signals, one from the source in the first phase and another one from the relay in the second phase. In this paper, we have evaluated the decoding performance at destination by using CP algorithm [7], that has lower complexity compared to the maximum a posteriori (MAP) decoding algorithm. CP algorithm is based on iterative decoding where two component decoders exchange information with each other during the decoding process.

B. Decoding at destination

Fig. 3 shows the proposed block diagram of the iterative decoding at destination, where $C_1$ and $C_2$ are the decoders for $C_1$ and $C_2$, respectively and both are using CP algorithm. The soft input of the decoder consists of the information from channel observation, $\mathbf{y}$ and feedback from the other decoder, $\mathbf{w}^t$. The decoding of the soft input is processed by using

![Block diagram of the proposed system](image-url)

Fig. 2. Block diagram of the proposed system

represents the location of the relay between the source and destination. Two phases are required to complete the transmission, during the first phase, the source broadcasts its information to both relay and destination. In the second phase, the relay process the received data from the source by using decode and forward relay protocol and transmits the processed data to destination while the source is in silent mode during this phase.

Fig. 2 shows the block diagram of the proposed model where $C_1$ and $C_2$ are the encoders for the block codes and in this paper, we have employed extended Bose, Chaudhuri, Hocquenghem (eBCH) codes as the block codes. $C_1$ and $C_2$ are assumed identical and has the same parameter set which is $(n, k, \delta)$ where $n$ is the codeword length, $k$ is the length of the information bit sequence and $\delta$ is the minimum Hamming distance of the codes. The information sequence $\mathbf{u}$ has length $k \times k$ and for each $k$ length, the information sequence is encoded by $C_1$ to produce codeword with length $n$. The encoded sequence $\mathbf{x}_S$ has length of $k \times n$ transmitted to the channel in signal form where bit 0 is mapped to $-1$ and bit 1 is mapped to 1. For simplicity, we consider additive white Gaussian noise (AWGN) channel for all channel links. The received signal at relay and destination are expressed as:

$$\mathbf{y}_{SD} = \mathbf{x}_S + \mathbf{z}_{SD},\quad (1)$$

$$\mathbf{y}_{SR} = \mathbf{x}_S + \mathbf{z}_{SR},\quad (2)$$

where $\mathbf{z}_{SD}$ and $\mathbf{z}_{SR}$ are the AWGN noise with zero mean and variance of $\sigma_{SD}^2$ and $\sigma_{SR}^2$ for source-destination link and source-relay link, respectively. Following the free space propagation model and assume equal transmit power at source and relay, the variance for $\sigma_{SR}^2$ can be expressed in terms of $\sigma_{SD}^2$ as:

$$\sigma_{SR}^2 = d^2 \sigma_{SD}^2,\quad (3)$$

where the corresponding signal-to-noise ratio (SNR) for source-destination link, $SNR_{SR}$ is given in terms of SNR source-destination link, $SNR_{SD}$ as:

$$SNR_{SR} = \frac{SNR_{SD}}{d^2}.\quad (4)$$

where $\mathbf{u}, \mathbf{y}, \mathbf{x}_S, \mathbf{z}_{SD}, \mathbf{z}_{SR}$
Chase 2 algorithm and the output is given in hard decision. Phyndiah in [7] has proposed calculation for the reliability of the hard decision output in order to make it suitable for iterative decoding. Given \( \mathbf{R} \) and \( \mathbf{D} \) are the soft input and hard decision output, respectively and \( \mathbf{D} \in \{ -1, 1 \} \), the soft output of the \( j \)-th element of \( \mathbf{D} \), \( d_j \) can be calculated as:

\[
 r'_j = \left( \frac{| \mathbf{R} - \mathbf{C} |^2 - | \mathbf{R} - \mathbf{D} |^2}{4} \right) d_j,
\]

where \( \mathbf{C} \) is the competing codeword with \( c_j \neq d_j \). In the case of \( \mathbf{C} \) is not found, the soft output can be determined by the following equation:

\[
 r'_j = \beta \times d_j \quad \text{with} \quad \beta \geq 0.
\]

The value of weighting factor \( \alpha \) and reliability factor \( \beta \) is pre-determined and in this paper, we have used the set value from [7] where the evolution of \( \alpha \) and \( \beta \) for every decoding step (half iteration) \( m \), are stated as follow:

\[
 \alpha(m) = [0, 0.2, 0.3, 0.5, 0.7, 0.9, 1, 1, 1, 1, 1, 1, 1],
\]

(10)

\[
 \beta(m) = [0.2, 0.4, 0.6, 0.8, 1, 1, 1, 1, 1, 1, 1, 1].
\]

The exchanging of information between both decoders are done by using extrinsic information \( \mathbf{w}' \) and only the systematic information part is exchanged. \( \mathbf{w}' \) is calculated by subtracting the soft output calculated in (9) with the soft input. In the case of no competing codeword, the extrinsic information is equal to the soft output calculated in (10). In order to reduce the dependency of \( \alpha \) on the turbo block codes, the mean absolute value of the extrinsic information calculated from (9) is normalized to one.

### III. Numerical Results

Simulations were conducted by using binary phase shift keying (BPSK) modulation over AWGN channel and eBCH(64,51,4) is used in the simulations as the default constituent codes of the turbo codes, with system code rate \( R_c = 0.66 \). The number of test patterns at destination is set to 16, where \( p = 4 \). Meanwhile in our study we have found out that the number of test patterns for Chase decoding at relay does not give significant difference to the overall performance, and therefore for simplicity, the \( p \) at relay is set to 0. For each simulation, at least \( 10^7 \) informations bits were transmitted in order to keep reasonable accuracy.

Fig. 4 shows the bit error rate (BER) performance of the proposed system when the relay is located halfway between the source and destination \( d = 0.5 \). The BER performance is evaluated with different numbers of decoding iteration at destination. The BER performance is improved as the number of iteration increases, but the improvement in terms of SNR at BER \( 10^{-5} \) is getting smaller and almost negligible beyond iteration 4. The finding is similar to the non cooperative case presented in [7], and also similar for all other locations of the relay, \( 0 \leq d \leq 1 \).

We have investigated the effect of \( p = 0, 1, 2, 3, 4, 5 \) at destination as shown in Fig. 5 for \( d = 0.5 \). The increase number of \( p \) corresponds to the increase in number of test pattern. Better improvement can be achieved as the number of test pattern increases but with the expense of increases in

![Fig. 3. Iterative decoding at destination by using Chase-Phyndiah algorithm at decoding step \( m \)](image)

![Fig. 4. BER performance of DTBC versus SNR SD with various iteration number for the turbo decoding at destination when the relay is located at \( d = 0.5 \)](image)

![Fig. 5. Comparison of BER performance of DTPC with various \( p \) values at destination after 4 iterations at \( d = 0.5 \)](image)
complexity. However, the improvement is getting smaller as \( p \) is getting higher, and therefore we have set \( p = 4 \) at destination as a good compromise between complexity and performance. It is worth to mention that the increase in number of test patterns at relay results in almost negligible improvement to the overall performance, since the overall performance is heavily depends on the direct link of source-destination.

Fig. 6 depicts the BER performance of the proposed system at various locations of relay. In the result, we have compared and shown that the performance of the proposed system is better than the non-cooperative system (point-to-point communications) [5]. When the relay location is 0.1L from the source, the improvement from the non-cooperative system is about 0.1dB at BER \( 10^{-5} \), and improvement of 0.25dB, 0.3dB, 0.35dB and 0.37dB for \( d = 0.2, 0.3, 0.4 \) and 0.5, respectively. Fig. 7 shows the BER performance from another perspective, whereby the BER performance is evaluated against the relay location, 0 \( \leq d \leq 1 \) at specific \( SNR_{SD} \). In this case we have demonstrated the BER performance at \( SNR_{SD} = 0.4dB, 0.8dB \) and 0.9dB. The best performance for \( SNR_{SD} = 0.4dB \) and 0.8dB is achieved when the relay is located in the middle between the source and destination, which is \( d = 0.5 \). However for \( SNR_{SD} = 0.9dB \), the best performance is achieved at \( d = 0.4 \). The improvement of the best performance of the proposed system from the non-cooperative system is getting larger as the error rate for the non-cooperative system gets lower. BER improvement of about \( 10^3 \) is achieved for \( SNR_{SD} = 0.4dB \), and for \( SNR_{SD} = 0.8dB \) and 0.9dB, improvement of about \( 10^2 \) and \( 10^3 \), respectively.

In order to investigate the effect of minimum Hamming distance, \( \delta \), in the proposed system, we have evaluated the BER performance when using constituent code of eBCH(64,57,4) with \( \delta = 4 \) and eBCH(64,45,8) with \( \delta = 8 \) as depicted in Fig. 8 and Fig. 9, respectively. The proposed DTBC system with constituent code of higher \( \delta \) achieves better performance and the improvement to the non-cooperative system is getting larger but with the trade off of reduction in code rate. It can be noted also, the location of the relay are the same for all cases when the best performance is achieved. When the non-cooperative system has error rate of about \( 10^{-2} \) and \( 10^{-3} \), the best performance of the proposed cooperative system is achieved at \( d = 0.5 \) and for error rate near to \( 10^{-4} \), the best performance is achieved at \( d = 0.4 \). The degradation in performance compared to the non-cooperative system starts to occur at \( d = 0.7 \) for non-cooperative system BER at \( 10^{-2} \), and as the BER for non-cooperative system is getting lower, the relay position for the degradation to happen is getting near to the source, \( d = 0.6 \) for non-cooperative system BER at about \( 10^{-4} \).
IV. Conclusion

In this paper, we have proposed and evaluated the performance of DTBC in a simple relay network based on line network model. The proposed scheme at destination use the iterative decoding technique and both decoder components for the turbo coding use soft-input soft-output CP algorithm. We have presented the performance of the proposed system with different iteration numbers at destination, different numbers of Chase test pattern at destination and the effect of relay location in line network model. From our results, we have shown that better performance is achieved by using the proposed cooperative scheme compared to the non-cooperative system as long as the relay location is not too distance from the source. Finally, we have demonstrated the effect of using different codes with different minimum Hamming distances. Future works include extension of this work into multi-sources relay networks.

References